Branching ratios of the decays of $\psi(3770)$ and $\Upsilon(10580)$ into light hadrons.

N. N. Achasov* and A. A. Kozhevnikov†
Laboratory of Theoretical Physics, S. L. Sobolev Institute for Mathematics, 630090, Novosibirsk, Russian Federation

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Taking into account the new data on the full width of $D^{*\pm}(2010)$ and the mass difference of the charged and neutral beauty mesons $B^{\pm}$, $B^0, \bar{B}^0$, the branching ratios of the decays $\psi(3770), \Upsilon(10580) \to \pi^+\pi^-$, $K\bar{K}$, $\rho(\omega)\pi$, $\rho(\omega)\eta$, $\rho(\omega)\eta'$, $K^*\bar{K}$ + c.c, $\rho^+\rho^-$, and $K^*\bar{K}^*$ are re-evaluated in the model in which the Okubo-Zweig-Iizuka rule is violated due to the real intermediate state $D\bar{D}$ in case of $\psi(3770)$ and $B\bar{B}$ in case of $\Upsilon(10580)$. The inclusive annihilation of $\psi(3770)$ and $\Upsilon(10580)$ into light hadrons is discussed.

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I. INTRODUCTION

Long ago we attracted attention to the decays of heavy quarkonia $\psi(3770)$ and $\Upsilon(10580)$ lying just above the corresponding open charm and beauty production thresholds, into the states consisting of light mesons [1]. Such decays are interesting from the point of view of the study of the two-step mechanism of the Okubo-Zweig-Iizuka (OZI) rule violation [2]. Recently interest in non-heavy-quark decays of heavy vector quarkonia was renewed [3].

As was argued in Ref. [1], the small branching ratios of the decays $J/\psi(1S) \to \omega\pi^0, \omega\eta, \omega\eta', \rho\pi, \rho\eta, \rho\eta', K^*\bar{K}$ + c.c., $\phi\eta, \phi\eta'$ and $\Upsilon(1S) \to \rho^0\pi^0, \pi^+\pi^-, K\bar{K}$ can be understood in the framework of the dispersion approach as the cancellation of the contributions of the intermediate states $D\bar{D}$, $D^*\bar{D} + c.c.$, $D^*\bar{D}^*$ and so on in the case of $J/\psi(1S)$ [or $B\bar{B}$, $B^*\bar{B} + c.c.$, $B^*\bar{B}^*$ and so on in the case of $\Upsilon(1S)$]. In the meantime, there is no reason for large suppression of the contribution of each specific intermediate state. The cancellation can be violated when new channel opens. Such a situation takes place, apparently, for the states $\psi(3770)$ and $\Upsilon(10580)$ whose decay into light hadrons proceeds via the real intermediate states $D^+D^- + D^0\bar{D}^0$ and $B^+B^- + B^0\bar{B}^0$, respectively. Hence, say, the $\psi(3770)$ decay amplitude to the pair of mesons M_1 and M_2 is approximated by its imaginary part which can be obtained from the unitarity condition,

$$\begin{aligned}\mathrm{Im}M_{\psi(3770)\to D\bar{D}\to M_1+M_2} &= \frac{1}{2(2\pi)^2}\int\frac{d^3q_D}{2E_D}\frac{d^3q_{\bar{D}}}{2E_{\bar{D}}}\delta^{(4)}(q_D+q_{\bar{D}}-q_{M_1}-q_{M_2})\times \\ &\quad M^*_{D\bar{D}\to M_1M_2}M_{\psi(3770)\to D\bar{D}},\end{aligned} \tag{1}$$

where

$$M_{\psi(3770)\to D\bar{D}} = g_{\psi(3770)D\bar{D}}\epsilon_\mu(q_D-q_{\bar{D}})_\mu,$$

ϵ_μ being the polarization four-vector of the $\psi(3770)$, and $M_{D\bar{D}\to M_1M_2}$ is the $D\bar{D} \to M_1M_2$ transition amplitude caused by the meson exchange with the nonzero heavy flavor quantum number. The case of $\Upsilon(10580)$ is obtained from Eq. (1) by the evident replacements. The necessary diagrams representing the amplitudes are given in Ref. [1].

Since, first, the new crucial data on the full width of $D^{*\pm}(2010)$ and on the mass difference of charged and neutral B mesons have appeared [4], second, there are the plans to study such decays of the $\psi(3770)$ meson with the upgraded CLEO-c facility [5], and, third, there is a huge number of the $\Upsilon(10580)$ mesons produced at BABAR [6] and BELLE [7] that exceeds 3×10^8, here, in Section II, we re-evaluate the branching fractions of the $\psi(3770)$ and $\Upsilon(10580)$ decay modes advertised above, taking into account the above progress of the experiment. The inclusive annihilation of $\psi(3770)$ and $\Upsilon(10580)$ into light hadrons via the mechanism under discussion is considered in Section III. Note that such a consideration was absent in Ref. [1]. In addition, the Coulomb interaction effects between D^+D^- and B^+B^- are discussed in Section IV. Section V is devoted to a brief discussion of the background problem and our hopes. Appendix contains some cumbersome technical details of calculation of the $\psi(3770)$ and $\Upsilon(10580)$ decays into two vector states, which were not adduced in Ref. [1].

*achasov@math.nsc.ru
†kozhev@math.nsc.ru;

II. EXCLUSIVE DECAYS

First, we give the expressions for imaginary part of the effective coupling constants of $\psi(3770)$ to the $\pi^+\pi^-$ and $\omega\pi^0$ states. They can be obtained upon using the following expressions for the amplitudes $M_{D^+D^-\to\pi^+\pi^-}$ and $M_{D^+D^-\to\omega\pi^0}$:

$$M_{D^+D^-\to\pi^+\pi^-} = g^2_{D^*D\pi^+}(q_{\pi^+}+q_{D^+},q_{\pi^-}+q_{D^-})\frac{\exp[\lambda_{D^*}(t-m^2_{D^{*0}})]}{m^2_{D^{*0}}-t}, \tag{2}$$

$$M_{D^+D^-\to\omega\pi^0} = 2g_{D^*D\omega}g_{D^*D\pi^0}\varepsilon_{\mu\nu\lambda\sigma}(q_\omega)_\mu\omega_\nu(q_{\pi^0})_\lambda(q_{D^-})_\sigma\frac{\exp[\lambda_{D^*}(t-m^2_{D^{*+}})]}{m^2_{D^{*+}}-t}, \tag{3}$$

where $\varepsilon_{\mu\nu\lambda\sigma}$ is totally antisymmetric unit tensor, $\varepsilon_{0123}=-1$, ω_ν is the ω meson polarization four vector, the four-momentum $(q_a)_\lambda$ is labelled by the name a of the corresponding particle, $t=(q_{D^+}-q_{\pi^+})^2$, and $(a,b)=a_0b_0-\boldsymbol{ab}$ denotes the scalar product of the two four-vectors $a=(a_0,\boldsymbol{a})$ and $b=(b_0,\boldsymbol{b})$. Note that the account is taken of the possibility of the non-point-like D^* exchange by means of introducing the form factor whose exponential form is parameterized by the slope λ_{D^*} to be specified below. The expressions for another isotopic state $D^0\bar{D}^0$ are obtained upon the evident replacements. The integration over the two-particle intermediate states is reduced to the integration over cosine of the angle $x=\cos\theta$ between the direction of the momenta of the intermediate D meson and the final meson M_1:

$$\frac{1}{(2\pi)^2}\int\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\delta^{(4)}(q-p_1-p_2)M^*_{D\bar{D}\to M_1M_2}M_{\psi(3770)\to D\bar{D}} = \frac{|\boldsymbol{p}_1|}{8\pi s^{1/2}}\int d\cos\theta\times M^*_{D\bar{D}\to M_1M_2}M_{\psi(3770)\to D\bar{D}}. \tag{4}$$

This results in the expressions for imaginary parts of the required coupling constants:

$$\begin{aligned}\mathrm{Im}g_{\psi(3770)\to\pi^+\pi^-} &= \frac{g_{\psi(3770)DD}g^2_{D^*D\pi^+}}{16\pi s^{1/2}q^2_{\pi\pi}}\left\{q_{D^+D^-}\left[s-m^2_{D^+}-m^2_\pi+\frac{1}{2}\left(m^2_{D^{*0}}+\right.\right.\right.\\ &\left.\left.\frac{(m^2_{D^+}-m^2_\pi)^2}{m^2_{D^{*0}}}\right)\right]\int_{-1}^{1}dx\frac{x\exp[2\lambda_{D^*}q_{D^+D^-}q_{\pi\pi}(b_{D^+D^-}+x)]}{b_{D^+D^-}+x}\mp\\ &\left.\text{contribution of } D^0\bar{D}^0 \text{ intermediate state}\right\}\approx\\ &-4g^2_{D^*D\pi^+}r_\mp\exp(-s\lambda_{D^*}/2),\end{aligned} \tag{5}$$

$$b_{D^+D^-} = \frac{2(m^2_{D^+}+m^2_\pi-m^2_{D^{*0}})-s}{4q_{D^+D^-}q_{\pi\pi}}, \tag{6}$$

and

$$\begin{aligned}\mathrm{Im}g_{\psi(3770)\to\omega\pi^0} &= -\frac{g_{\psi(3770)DD}g_{D^*D\omega}g_{D^*D\pi^0}}{8\pi s^{1/2}q_{\omega\pi}}\left\{q^2_{D^+D^-}\int_{-1}^{1}dx\frac{1-x^2}{a_{D^+D^-}+x}\times\right.\\ &\exp[2\lambda_{D^*}q_{D^+D^-}q_{\omega\pi}(a_{D^+D^-}+x)]\mp\\ &\left.\text{contribution of } D^0\bar{D}^0 \text{ intermediate state}\right\}\approx\\ &4g_{D^*D\omega}g_{D^*D\pi^0}r_\mp\exp(-s\lambda_{D^*}/2),\end{aligned} \tag{7}$$

$$a_{D^+D^-} = \frac{m^2_{D^+}+m^2_\omega-m^2_{D^{*+}}-s^{1/2}E_\omega}{2q_{D^+D^-}q_{\omega\pi}}. \tag{8}$$

Hereafter $s^{1/2}=m_{\psi(3770)}$ $[m_{\Upsilon(10580)}]$ when discussing the decays of $\psi(3770)$ $[\Upsilon(10580)]$, respectively. The contribution of the $D^0\bar{D}^0$ intermediate state can be obtained from the D^+D^- one in Eqs. (5), (6), (7), and (8) by the evident replacements. In the above formulas, the approximate expressions containing $r_\mp$, where

$$r_\mp = \frac{g_{\psi(3770)DD}}{6\pi m^3_{\psi(3770)}}\cdot(q^3_{D^+D^-}\mp q^3_{D^0\bar{D}^0}) \tag{9}$$

are valid near threshold assuming the approximate degeneracy of D^* and D mesons [1]. The sign plus (minus) corresponds to the situation when isospin is conserved (violated) in the course of decay,

$$\begin{aligned} E_b &= \frac{1}{2m_a}(m_a^2 + m_b^2 - m_c^2), \\ q_{bc} &= \frac{1}{2m_a} \times \sqrt{[m_a^2 - (m_b - m_c)^2] \times [m_a^2 - (m_b + m_c)^2]} \end{aligned} \quad (10)$$

are, respectively, the center-of-mass energy of the particle b, the momentum of the particle b (or c) in the final state of the decay $a \to b + c$ expressed through the masses $m_{a,b,c}$. The necessary coupling constants are evaluated as

$$|g_{\psi(3770)D\bar{D}}| = \left[\frac{6\pi m^2_{\psi(3770)}\Gamma_{\psi(3770)\to D^+D^- + D^0\bar{D}^0}}{q^3_{D^+D^-} + q^3_{D^0\bar{D}^0}}\right]^{1/2} = 13.4, \quad (11)$$

$$|g_{D^*D\pi^+}| = \left[\frac{6\pi\Gamma_{D^{*\pm}}\left(B_{D^{*\pm}\to D^0\pi^\pm} + B_{D^{*\pm}\to D^\pm\pi^0}\right)}{q^3_{D^0\pi^\pm} + \frac{1}{2}q^3_{D^\pm\pi^0}}\right]^{1/2} = 9.1, \quad (12)$$

where $g_{D^*D\pi^0} = \frac{1}{\sqrt{2}}g_{D^*D\pi^+}$. [Compare 9.1 in the right hand side of Eq. (12) with the figure of 4.5 resulting from the quark counting rule relation $|g_{D^*D\pi^+}| \approx |g_{K^*K\pi^+}|$.] The coupling constant $g_{D^*D\omega}$ cannot be found from existing data. We choose the quark counting rule to obtain its magnitude:

$$g_{D^*D\omega} \approx g_{K^*K\omega} \approx \frac{1}{2}g_{\omega\rho\pi} = 7.2\ \mathrm{GeV}^{-1}, \quad (13)$$

where $g_{\omega\rho\pi} = 14.3\ \mathrm{GeV}^{-1}$ is obtained from the branching ratio $B_{\omega\to\pi^+\pi^-\pi^0}$. Another relations necessary for obtaining the rates of the decays $\psi(3770) \to \omega\eta, \omega\eta', \rho\pi, \rho\eta, \rho\eta'$ from $B_{\psi(3770)\to\omega\pi^0}$ are

$$g_{D^*D\eta} \approx -\sqrt{\frac{2}{3}}g_{D^*D\pi^0} \approx \sqrt{2}g_{D^*D\eta'}, \quad g_{D^{*0}D^0\omega} = -g_{D^{*0}D^0\rho} = g_{D^{*+}D^-\rho}, \quad (14)$$

where the quark content $\eta = \sqrt{\frac{1}{3}}(u\bar{u}+d\bar{d}-s\bar{s})$, $\eta' = \sqrt{\frac{1}{6}}(u\bar{u}+d\bar{d}+2s\bar{s})$ corresponding to the pseudoscalar mixing angle $\theta_P = -19.5°$ is understood. To obtain $\mathrm{Im}g_{\psi(3770)\to\bar{K}^0K^0}$ [$\mathrm{Im}g_{\psi(3770)\to K^-K^+}$] from $\mathrm{Im}g_{\psi(3770)\to D\bar{D}\to\pi^+\pi^-}$, one should take into account the unique intermediate state D^+D^- ($D^0\bar{D}^0$) and the D_s^* exchange in the amplitude $D^+D^- \to \bar{K}^0K^0$ ($D^0\bar{D}^0 \to K^-K^+$), and make the proper replacement in the coupling constants. The necessary relation is

$$g_{D_s^{*+}D^+K^0} = g_{D^{*+}D^0\pi^+}. \quad (15)$$

$\mathrm{Im}g_{\psi(3770)\to D\bar{D}\to\bar{K}^*K}$ can be found from $\mathrm{Im}g_{\psi(3770)\to D\bar{D}\to\omega\pi^0}$ in the same manner as in the above case of the pseudoscalar final state. The necessary relations are

$$g_{D_s^*DK^*} \approx \sqrt{2}g_{D^*D^*\pi^0} \approx \sqrt{2}g_{K^*K^*\pi^0} \approx \frac{g_{\omega\rho\pi}}{\sqrt{2}}. \quad (16)$$

The partial widths of the decays to the pair of pseudoscalars PP and vector+pseudoscalar $V + P$ are

$$\Gamma_{\psi(3770)\to PP} = \frac{|g_{\psi(3770)PP}|^2}{6\pi m^2_{\psi(3770)}}q^3_{PP},\ \Gamma_{\psi(3770)\to VP} = \frac{|g_{\psi(3770)VP}|^2}{12\pi}q^3_{VP}. \quad (17)$$

The adopted approximation permits one to take $q_{PP} \approx q_{VP} \approx \frac{1}{2}m_{\psi(3770)}$.

The amplitude of the decay $\psi(3770) \to \rho^+\rho^-, K^*\bar{K}^*$ contains four independent structures:

$$\begin{aligned} M(V \to V_1V_2) &= \frac{1}{2}g_1(\epsilon^{(V)}, q_1 - q_2)(\epsilon^{(V_1)}, \epsilon^{(V_2)}) + g_2(\epsilon^{(V_2)}, q_1)(\epsilon^{(V)}, \epsilon^{(V_1)}) + \\ &\quad g_3(\epsilon^{(V_1)}, q_2)(\epsilon^{(V)}, \epsilon^{(V_2)}) + \\ &\quad \frac{1}{2}g_4(\epsilon^{(V)}, q_1 - q_2)(\epsilon^{(V_1)}, q_2)(\epsilon^{(V_2)}), q_1), \end{aligned} \quad (18)$$

where $\epsilon^{(V_{1,2})}, q_{1,2}$ stands for the polarization and momentum four-vectors of the corresponding vector particle. The partial width is

$$\Gamma_{\psi(3770)\to\rho^+\rho^-}(s) = \frac{q^3_{\rho\rho}}{24\pi s}\left\{2\left[|g_1|^2 + (|g_2|^2+|g_3|^2)\frac{s}{m^2_\rho}\right] + |g_1 + (g_2-g_3)\frac{s^{1/2}}{m_\rho} + Gq^2_{\rho\rho}|^2\right\}, \tag{19}$$

where $s = m^2_{\psi(3770)}$,

$$G = \frac{1}{m^2_\rho}\left[2g_1 + \frac{2s^{1/2}(g_2-g_3)}{(s^{1/2}+2m_\rho)} + g_4 s\right]. \tag{20}$$

The derivation of the expressions for imaginary parts of the coupling constants $g_{1,2,3}$ and G is outlined in Appendix. The results for the $K^{*-}K^{*+}$ ($\bar{K}^{*0}K^{*0}$) final state are obtained from the $\rho^+\rho^-$ one by taking the unique $D^0\bar{D}^0$ (D^+D^-) intermediate state contribution to the unitarity relation Eq. (1) and allowing for the exchanges D_s and D^*_s in the $D\bar{D}\to\bar{K}^*K^*$ reaction, and by using the quark counting rule relations $|g_{\bar{K}^{*0}D^+D^-_s}| = \sqrt{2}|g_{\rho^0 DD}|$, $|g_{\bar{K}^{*0}D^+D^{*-}_s}| = \sqrt{2}|g_{D^*D\rho^0}|$.

We use $\lambda_D \approx \lambda_{D^*} = 0.27$ GeV^{-2} and $\lambda_B \approx \lambda_{B^*} = 0.04$ GeV^{-2}, that is, as before in Ref. [1] we expect that $\lambda_{D,D^*} \sim 1/m^2_{D^*}$ and $\lambda_{B,B^*} \sim 1/m^2_{B^*}$. In addition, as before in Ref. [1], we take into account the two-fold suppression of the amplitudes due to the absorption in the final state. As a result, the branching ratios are suppressed by factor 30 in comparison with the ones calculated in the model with the point like exchange. If to take courage for use the Regge exchange [8] at low energy, $\ln(s/s_0) \approx 1$, one get the branching ratios nearly our ones. So we hope that our estimations are conservative enough.

The resulting branching ratios, together with the expected number of events adopted to the recent or planned CLEO integral luminosity [5], and the ratio of the signal to the exclusive background estimated in the simple vector dominance model are collected in Table I. Note that the absorption suppression factor 1/4 is excluded in the case of the final states containing J/ψ meson because both the intermediate and final states in the reaction $D\bar{D}\to J/\psi+\pi^0(\eta)$ are in the threshold region.

Almost all said above about $\psi(3770)$ can be translated to the case of $\Upsilon(10580)\equiv\Upsilon(4S)$ meson decays to light mesons by means of the replacements $\psi(3770)\to\Upsilon(10580), c\to b, D\to B, D^*_s\to B^*_s$ etc. The suppression due to the vertex form factor and the absorption in the final state is assumed to be the same as in the case of $\psi(3770)$. The difference is that the electromagnetic mass difference of the vector B^{*+} and B^{*0} mesons is not known. Our assumption is the above mass difference is vanishing. Using the quark model relations expressing all unknown coupling constants through the known ones which are analogous to those used in the case of the $\psi(3770)$ decays, one can evaluate the branching fractions of the $\Upsilon(10580)$ decays. The results of this evaluation, again taking into account the still unruled possibility of the isospin $I=1$ are given in the table II. Note that the absorption suppression factor 1/4 is excluded in the case of the final states containing $\Upsilon(1S)$ meson because both the intermediate and final states in the reaction $B\bar{B}\to\Upsilon(1S)+\pi^0(\eta,\eta')$ are in the threshold region. Also given is the ratio of the signal to the exclusive background estimated in the simple vector dominance model.

III. INCLUSIVE DECAYS

Let us discuss the inclusive annihilation $\psi(3770)$ and $\Upsilon(10580)$ into light hadrons, $\psi(3770)\to D^+D^-+D^0\overline{D^0}\to X$ and $\Upsilon(10580)\to B^+B^-+B^0\overline{B^0}\to X$. Supposing that the isotopic symmetry is broken only by the mass difference of charge and neutral particles in an isotopic multiplet and by electromagnetic interaction between D^+D^- and B^+B^-, we can express the total branching ratios of the decays of $\psi(3770)$ and $\Upsilon(10580)$ into light hadrons X via the branching ratios of the $\psi(3770)\to D\overline{D}$ and $\Upsilon(10580)\to B\overline{B}$ decays and the annihilation cross-sections of the $D\overline{D}$ and $B\overline{B}$ real intermediate states into light hadrons X

$$\sum_X B_{V\to P^+P^-+P^0\overline{P^0}\to X} = \left(m^2_V v^2_P/48\pi\right)\times B_{V\to P^0\overline{P^0}}\times \sum_X\left[1+(-1)^{I_X+I_V}|c_{P^\pm}|^2\times\left(v_{P^\pm}/v_{P^0}\right)^3\right]^2\times\sigma_P\left(P^0\overline{P^0}\to X\right), \tag{21}$$

TABLE I: Branching ratios of the $\psi(3770)$ decays into the pair of light mesons. Also estimated is the expected number of events $N_{1,2}$, where 1 and 2 correspond to the integral luminosity $\int \mathcal{L}dt = 60.3\text{pb}^{-1}$ and 3fb^{-1} attained and planned at CLEO-c[5]. The quantities without (with) parentheses correspond to the isospin $I = 0$ taking place in the model of $c\bar{c}$ quarkonium or $D\bar{D}$ molecule/four-quark state with zero isospin ($D\bar{D}$ molecule/four-quark isovector state). The charged and neutral pairs of strange particles have coincident branching ratios for both values of isospin.

mode f	$B_{\psi(3770)\to f}$	N_1	N_2	signal/background
$\pi^+\pi^-$	$3\times10^{-5}(7\times10^{-4})$	20 (490)	1000 (24500)	0.1 (2)
K^+K^-	9×10^{-5}	60	3000	0.2
$K^0\bar{K}^0$	9×10^{-5}	60	3000	5
$\omega\pi^0$	$6\times10^{-5}(2\times10^{-3})$	40 (1400)	2000 (70000)	0.004 (0.1)
$\omega\eta$	$1\times10^{-3}(4\times10^{-5})$	700 (30)	35000 (1550)	2 (0.1)
$\omega\eta'$	$5\times10^{-4}(2\times10^{-5})$	350 (10)	17500 (500)	2 (0.1)
$\rho\pi$	$5\times10^{-3}(2\times10^{-4})$	3500 (140)	175000 (7000)	1 (0.05)
$\rho\eta$	$4\times10^{-5}(1\times10^{-3})$	30 (700)	1500 (35000)	0.01 (0.4)
$\rho\eta'$	$2\times10^{-5}(5\times10^{-4})$	15 (350)	750 (17500)	0.01 (0.4)
$K^{*+}K^- +$ c.c	8×10^{-4}	560	28000	0.5
$K^{*0}\bar{K}^0 +$ c.c	8×10^{-4}	560	28000	0.05
$\rho^+\rho^-$	$7\times10^{-5}(2\times10^{-3})$	40 (1400)	2000 (70000)	0.003 (0.007)
$K^{*+}K^{*-}$	4×10^{-4}	270	14000	0.01
$K^{*0}\bar{K}^{*0}$	4×10^{-4}	270	14000	0.3
$J/\psi+\pi^0$	2×10^{-5} (5×10^{-4})	15 (350)	750 (17500)	–
$J/\psi+\eta$	8×10^{-5} (3×10^{-6})	60 (2)	3000 (100)	–
$\sum_f B_{\psi(3770)\to f}$	0.009 (0.009)	~ 6000 (~ 6000)	$\sim 3\times10^5(\sim 3\times10^5)$	
$B_{\psi(3770)\to 3\text{gluons}}$	2×10^{-4}	–	–	

TABLE II: Branching ratios of the $\Upsilon(10580)$ decays into the pair of light mesons. Also estimated is the expected number of events N corresponding to the integral luminosity $\int \mathcal{L}dt = 81.7\text{fb}^{-1}$ attained at BABAR [6]. The number of events expected in the whole statistics $\int \mathcal{L}dt = 140\text{fb}^{-1}$ at BELLE [7] is obtained by multiplying N by the factor 1.7. The quantities without (with) parentheses correspond to the isospin $I = 0$ taking place in the model of $b\bar{b}$ quarkonium or $B\bar{B}$ molecule/four-quark state with zero isospin ($B\bar{B}$ molecule/four-quark isovector state). The charged and neutral pairs of strange particles have coincident branching ratios for both values of isospin.

mode f	$B_{\Upsilon(10580)\to f}$	N	signal/background
$\pi^+\pi^-$	7×10^{-8} (9×10^{-5})	20 (2.7×10^4)	0.03 (40)
K^+K^-	2×10^{-5}	6000	6
$K^0\bar{K}^0$	2×10^{-5}	6000	150
$\omega\pi^0$	1×10^{-6} (1×10^{-3})	300 (3×10^5)	0.001 (2)
$\omega\eta$	9×10^{-4} (7×10^{-7})	3×10^5 (240)	40 (0.03)
$\omega\eta'$	5×10^{-4} (4×10^{-7})	1.5×10^5 (120)	40 (0.03)
$\rho\pi$	4×10^{-3} (3×10^{-6})	1.5×10^6 (1200)	30 (0.02)
$\rho\eta$	7×10^{-7} (1×10^{-3})	240 (3×10^5)	0.006 (7)
$\rho\eta'$	4×10^{-7} (5×10^{-4})	120 (1.5×10^5)	0.006 (7)
$K^{*+}K^- +$ c.c	1×10^{-3}	3×10^5	18
$K^{*0}\bar{K}^0 +$ c.c	1×10^{-3}	3×10^5	2
$\rho^+\rho^-$	6×10^{-6} (7×10^{-3})	1500 (2×10^6)	0.004 (7)
$K^{*+}K^{*-}$	2×10^{-3}	6×10^5	1
$K^{*0}\bar{K}^{*0}$	2×10^{-3}	5×10^5	25
$\Upsilon(10580)+\pi^0$	1×10^{-7} (1×10^{-4})	30 (30000)	–
$\Upsilon(10580)+\eta$	6×10^{-5} (4×10^{-8})	18000 (10)	–
$\Upsilon(10580)+\eta'$	6×10^{-6} (5×10^{-9})	1800 (1)	–
$\sum_f B_{\Upsilon(10580)\to f}$	0.012 (0.016)	$\sim 4\times10^6$ $(\sim 5\times10^6)$	–
$B_{\Upsilon(10580)\to 3\text{gluons}}$	6×10^{-4}	–	–

where $V = \psi(3770)$ (or $\Upsilon(10580)$); $P(\overline{P}) = D(\overline{D})$ (or $B(\overline{B})$); where v_P is velocity of P(or $\overline{P}$) in c.m. system; I_X and I_V are the isotopic spins of X and V, respectively; $c_{P^\pm}$ takes into account electromagnetic interaction between P^+ and P^-; $\sum_X \sigma_P \left(P^0\overline{P^0} \to X\right)$ is the total annihilation cross-section $P^0\overline{P^0} \to X$ in the P wave with I_X=0, or 1. When deriving Eq. (21) terms of order $\left(v^n_{P^\pm} - v^n_{P^0}\right)$ are neglected in

$$\sum_X \sigma_P \left(P^+P^- \to X\right) / \sum_X \sigma_P \left(P^0\overline{P^0} \to X\right) , \tag{22}$$

where n=1, 2, ...; $v_{D^\pm} \approx 0.128$ and $v_{D^0} \approx 0.147$; $v_{B^\pm} \approx 0.064$ and $v_{B^0} \approx 0.063$.

The factors of suppression of the foreign isotopic spin $(-1)^{I_X+I_V} = -1$ in Eq. (21)

$$d = \left[1 - |c_{D^\pm}|^2 \times (v_{D^\pm}/v_{D^0})^3\right]^2 / \left[1 + |c_{D^\pm}|^2 \times (v_{D^\pm}/v_{D^0})^3\right]^2 \approx 0.04 \tag{23}$$

and

$$b = \left[1 - |c_{B^\pm}|^2 \times (v_{B^\pm}/v_{B^0})^3\right]^2 / \left[1 + |c_{B^\pm}|^2 \times (v_{B^\pm}/v_{B^0})^3\right]^2 \approx 0.0008 \tag{24}$$

at $c_{P^\pm} = 1$.

So, measuring the total branching ratios of the decays of $\psi(3770)$ and $\Upsilon(10580)$ into light hadrons gives information about the P wave annihilation cross-sections of $D\overline{D}$ and $B\overline{B}$ into light hadrons with the proper isotopic spin $(-1)^{I_X+I_V} = 1$. At present experiment allows to the value of $\sum_X B_{\psi(3770)\to X}$ to be up to 10% [9].

$$\sum_X B_{\psi(3770)\to D^+D^- + D^0\overline{D^0}\to X} = 1\% \quad \text{and} \quad \sum_X B_{\Upsilon(10580)\to B^+B^- + B^0\overline{B^0}\to X} = 1\% \tag{25}$$

correspond

$$\sum_X \sigma_P(D^0\overline{D^0} \to X) \approx 1.5\,\text{mb} \quad \text{and} \quad \sum_X \sigma_P(B^0\overline{B^0} \to X) \approx 0.64\,\text{mb}\,, \tag{26}$$

respectively. Note that $\sum_X \sigma_P(D^0\overline{D^0} \to X) \sim v_{D^0}$ and $\sum_X \sigma_P(B^0\overline{B^0} \to X) \sim v_{B^0}$.

IV. COULOMB CORRECTIONS

Current experiment [10, 11] allows to draw a conclusion about the electromagnetic corrections (the $|c_{P^\pm}|^2$ factors). CLEO gives

$$r^{ex}_D = \sigma(e^+e^- \to D^+D^-)/\sigma(e^+e^- \to D^0\overline{D^0}) = 0.776 \pm 0.024^{+0.014}_{-0.006} \tag{27}$$

at $E_{cm} = 3773$ MeV. The theoretical prediction at $c_{D^\pm} = 1$ is

$$r^{th}_D = 0.6913 \pm 0.0085\,. \tag{28}$$

So, experiment [10] gives

$$|c^{ex}_{D^\pm}|^2 = r^{ex}_D / r^{th}_D = 1.123 \pm 0.033^{+0.02}_{-0.01}\,, \tag{29}$$

that does not contradict the point like electromagnetic correction [12]

$$|c^{th}_{D^\pm}|^2 \approx 1 + \frac{\alpha\pi}{2v_{D^\pm}} = 1.086 \pm 0.001\,. \tag{30}$$

Note that Eq. (29) leads to the additional suppression in Eq. (23): $d \to 0.02$. As for $c_{B^\pm}$, one can get an information about this in indirect way from BABAR [11]

$$B_{\Upsilon(10580)\to B^0\overline{B^0}} = 0.487 \pm 0.010(stat) \pm 0.008(sys)\,. \tag{31}$$

Supposing the two channel dominance $\Upsilon(10580) \to B^+B^-\,,\ B^0\overline{B^0}$, one can get the theoretical predictions

$$B^{1th}_{\Upsilon(10580)\to B^0\overline{B^0}} = 0.489 \pm 0.010 \tag{32}$$

at $c^{1th}_{B^\pm}=1$ and

$$B^{2th}_{\Upsilon(10580)\to B^0\overline{B^0}}=0.448\pm0.010 \tag{33}$$

at the point like electromagnetic correction

$$|c^{2th}_{B^\pm}|^2\approx 1+\frac{\alpha\pi}{2v_{B^\pm}}=1.178\pm0.004\,. \tag{34}$$

So, there is good agreement between the data Eq. (31) and the calculation without an electromagnetic correction Eq. (32), at the same time there are three standard deviations between the experiment value Eq. (31) and the calculation taking into account the point like electromagnetic interaction Eq. (33). This can be considered as an evidence for the role of the $B^+(B^-)$ structure [13] (or violation of isotopic symmetry for the coupling constants $g_{\Upsilon(10580)D^+D^-}$ and $g_{\Upsilon(10580)D^0\overline{D^0}}$). The factor $|c_{B^\pm}|^2=1.08$ would give $B_{\Upsilon(10580)\to B^0\overline{B^0}}=0.469$ consistent with experiment within one standard deviation. Note that $|c_{B^\pm}|^2=1.08$ leads to the increase in Eq. (24): $b\to0.0044$. Whereas Eq. (34) would lead to the increase in Eq. (24): $b\to0.0119$.

Note that the correction factors Eq. (30) and (34) result, respectively, in suppressing by the factor of 0.6 of the $\psi(3770)$ decays with foreign isospin Table I, which is beyond the accuracy of the calculation, and in enhancing by the factor of 17 of the analogous processes of the $\Upsilon(10580)$ decays in Table II. This follows from the replacement of the factor $r_\mp$ Eq. (9) by the modified factor

$$r_\mp^{\text{modified}}=\frac{g_{\psi(3770)DD}}{6\pi m^3_{\psi(3770)}}\cdot\left(q^3_{D^+D^-}\times|c^{th}_{D^\pm}|^2\mp q^3_{D^0\bar D^0}\right) \tag{35}$$

and the analogous replacement in the case of $\Upsilon(10580)$.

V. CONCLUSION

The inclusive background at $\psi(3770)$ is

$$\sigma\left(e^+e^-\to u\bar u+d\bar d+s\bar s\,,\ \sqrt{s}=3770\,\text{MeV}\right)\approx1.22\cdot10^{-32}\,\text{cm}^2\,, \tag{36}$$

the signal is

$$\sum_X\sigma\left(e^+e^-\to\psi(3770)\to X\right)\approx1.19\cdot B_{\psi(3770)\to X}\cdot10^{-32}\,\text{cm}^2\,. \tag{37}$$

The inclusive background at $\Upsilon(10580)$ is

$$\sigma\left(e^+e^-\to u\bar u+d\bar d+s\bar s\,,\ \sqrt{s}=10580\,\text{MeV}\right)\approx1.55\cdot10^{-33}\,\text{cm}^2\,, \tag{38}$$

the signal is

$$\sum_X\sigma\left(e^+e^-\to\Upsilon(10580)\to X\right)\approx3.77\cdot B_{\Upsilon(10580)\to X}\cdot10^{-33}\,\text{cm}^2\,. \tag{39}$$

So, the relation signal/background is better in exclusive decays in many cases, compare the tables I and II with the ratios of Eq. (37) to Eq. (36) and Eq. (39) to Eq. (38).

The exclusive background is estimated as an incoherent one using the simple vector dominance model, see the tables I and II. Certainly, the problem of the background should be discovered only in the way. Nevertheless it is clear even now that the exclusive background is overvalued by a factor three in the $\omega\pi^0$ mode and by a factor ten in the $\rho^0\eta$ mode, see [14]. Note that interference effects can expand experimental possibilities.

So, the study of non-heavy-quark decays of heavy vector quarkonia promises the exciting cruise unless discovery a new land (four-quark nature of $\psi(3770)$ or $\Upsilon(10580)$, for example). But in any case Foundation of Low Energy Physics of Charmed and Beauty hadrons will be laid.

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APPENDIX

Here we outline the derivation of the contributions to the imaginary parts of the coupling constants $g_{1,2,3}$ and G standing in the $V \to V\bar{V}$ decay amplitude, where $V = \psi(3770)$, $\Upsilon(10580)$, and $V = \rho^+, K^{*+}, K^{*0}$. See Eqs. (18) and (20). To be specific, let us choose the final state $\rho^+\rho^-$ to discuss the details of the derivation. There are two contributions to the amplitude of the reaction $D^+D^- \to \rho^+\rho^-$ due to the D and D^* exchanges. They are:

$$\begin{aligned} M^{(D)}_{D^+D^-\to\rho^+\rho^-} &= 4g^2_{\rho^+D^+D^0}\frac{(q_{D^+})_\mu(q_{D^-})_\nu\rho^+_\mu\rho^-_\nu}{m^2_{D^0}-t}\exp[\lambda_D(t-m^2_{D^0})], \\ M^{(D^*)}_{D^+D^-\to\rho^+\rho^-} &= g^2_{\rho^+D^+D^{*0}}\varepsilon_{\mu\nu\lambda\sigma}\varepsilon_{\mu'\nu\lambda'\sigma'}k_\mu k_{\mu'}(q_{D^+})_\lambda(q_{D^-})_{\lambda'}\rho^+_\sigma\rho^-_{\sigma'}\times \\ &\quad \frac{\exp[\lambda_{D^*}(t-m^2_{D^{*0}})]}{m^2_{D^{*0}}-t}, \end{aligned} \tag{A.1}$$

where $k = q_{D^+} - q_{\rho^+}$, $t = k^2$, and $\rho^\pm_\sigma$ stands for the polarization four-vector of the final $\rho^\pm$ meson. The expressions for the amplitude of the reaction $D^0\bar{D}^0 \to \rho^+\rho^-$ are obtained from Eq. (A.1) by evident replacements. When finding the contributions to the imaginary part of the $\psi(3770) \to \rho^+\rho^-$ decay amplitude, one should express the polarization four-vectors of moving $\rho^\pm$ mesons through the polarization three-vectors $\boldsymbol{\xi}^{\rho^\pm}$ in their respective rest frames:

$$(\rho^\pm_0, \boldsymbol{\rho}^\pm) = \left(\frac{(\boldsymbol{q}^\pm\boldsymbol{\xi}^{\rho^\pm})}{m_\rho}, \boldsymbol{\xi}^{\rho^\pm} + \frac{\boldsymbol{q}^\pm(\boldsymbol{q}^\pm\boldsymbol{\xi}^{\rho^\pm})}{m_\rho(q^\pm_0+m_\rho)}\right), \tag{A.2}$$

where $(q_{\rho^\pm})_\mu \equiv q^\pm_\mu = (q^\pm_0, \mathbf{q}^\pm)$. Then the $V \to \rho^+\rho^-$ decay amplitude Eq. (18) in the rest frame of the decaying vector meson V can be written as

$$\begin{aligned} M_{V\to\rho^+\rho^-} = g_1(\boldsymbol{\xi}\boldsymbol{q}^+)(\boldsymbol{\xi}^{\rho^+}\boldsymbol{\xi}^{\rho^-}) + g_2\frac{s^{1/2}}{m_\rho}(\boldsymbol{\xi}^-\boldsymbol{q}^+)(\boldsymbol{\xi}^{\rho^+}\boldsymbol{\xi}) - g_3\frac{s^{1/2}}{m_\rho}(\boldsymbol{\xi}^+\boldsymbol{q}^+)(\boldsymbol{\xi}^{\rho^-}\boldsymbol{\xi}) +& \\ G(\boldsymbol{\xi}\boldsymbol{q}^+)(\boldsymbol{\xi}^+\boldsymbol{q}^+)(\boldsymbol{\xi}^-\boldsymbol{q}^+),& \end{aligned} \tag{A.3}$$

where $\boldsymbol{\xi}$ is the polarization three-vector of the decaying V, and G is given by Eq. (20). The results of integration over intermediate state D^+D^- can be represented as the sum of contributions of the D and D^* exchanges $g_{1,2,3} = g^{(D)}_{1,2,3} + g^{(D^*)}_{1,2,3}$, $G = G^{(D)} + G^{(D^*)}$, where

$$\begin{aligned} g^{(D)}_1 &= N_DA_6, \\ g^{(D)}_2 &= -g^{(D)}_3 = -\frac{N_D}{s^{1/2}(s^{1/2}+2m_\rho)}\left[A_2 + m_\rho(s^{1/2}+m_\rho)A_5 - m_\rho(s^{1/2}+2m_\rho)A_6\right], \\ G^{(D)} &= \frac{N_D}{[m_\rho(s^{1/2}+2m_\rho)]^2}\left\{A_3 + (2A_1+A_4)m_\rho(s^{1/2}+m_\rho) - \frac{2m_\rho(s^{1/2}+2m_\rho)}{(\boldsymbol{q}^+)^2}\times\right. \\ &\quad \left.[B_2 + B_5m_\rho(s^{1/2}+m_\rho) + \frac{1}{2}B_6m_\rho(s^{1/2}+2m_\rho)]\right\}, \\ g^{(D^*)}_1 &= N_{D^*}\left\{-\left[m^2_{D^*}\left(\frac{s}{2}-m^2_\rho\right) + \frac{1}{4}\left(m^2_{D^*}+m^2_\rho-m^2_D\right)\right]A_4 + \frac{1}{2}\left(s - m^2_\rho + m^2_{D^*} - \right.\right. \\ &\quad \left.\left. m^2_D\right)A_1 - \frac{1}{4}A_3 + \left(\frac{s}{2}-m^2_\rho\right)A_6\right\}, \\ g^{(D^*)}_2 &= -g^{(D^*)}_3 = \frac{m_\rho N_{D^*}}{s^{1/2}}\left\{\left[\left(\frac{s}{2}-m^2_\rho\right)\frac{s^{1/2}+m_\rho}{s^{1/2}+2m_\rho} + \frac{s^{1/2}}{2m_\rho}(m^2_{D^*}+m^2_\rho-m^2_D)\right]A_5 - \right. \\ &\quad \left.\frac{s^{1/2}+m_\rho}{s^{1/2}+2m_\rho}A_2 - \left(\frac{s}{2}-m^2_\rho\right)A_6\right\}, \\ G^{(D^*)} &= N_{D^*}\left\{A_4\left[2m^2_{D^*} - \frac{s^{1/2}(s^{1/2}+m_\rho)}{m_\rho(s^{1/2}+2m_\rho)}(m^2_{D^*}+m^2_\rho-m^2_D) - \left(\frac{s}{2}-m^2_\rho\right)\times\right.\right. \\ &\quad \left.\left.\left(\frac{s^{1/2}+m_\rho}{s^{1/2}+2m_\rho}\right)^2 - \frac{1}{2m^2_\rho}(m^2_{D^*}+m^2_\rho-m^2_D)^2\right] - \frac{A_3}{(s^{1/2}+2m_\rho)^2} + 2A_1 + \frac{B_5}{(\boldsymbol{q}^+)^2}\times\right. \end{aligned}$$

$$\left[(s-2m_\rho^2)\frac{s^{1/2}+m_\rho}{s^{1/2}+2m_\rho}+\frac{s^{1/2}}{m_\rho}(m_{D^*}^2+m_\rho^2-m_D^2)\right]-\frac{2B_2}{(\boldsymbol{q}^+)^2}\times\frac{s^{1/2}+m_\rho}{s^{1/2}+2m_\rho}-$$
$$\left.\left(\frac{s^{1/2}}{2}-m_\rho^2\right)\frac{B_6}{(\boldsymbol{q}^+)^2}\right\}. \tag{A.4}$$

The normalization factors are

$$\begin{aligned} N_D &= 8g_{\psi(3770)D\bar{D}}g^2_{\rho^0 D\bar{D}}, \\ N_{D^*} &= -g_{\psi(3770)D\bar{D}}g^2_{\rho^0 D^*\bar{D}}, \end{aligned} \tag{A.5}$$

where the quark model relations $|g_{\rho^0 D^*\bar{D}}| \approx \frac{1}{2}g_{\omega\rho\pi} \approx 7.2\ \mathrm{GeV}^{-1}$, $|g_{\rho^0 D\bar{D}}| \approx |g_{\rho^0 KK}| = \frac{1}{\sqrt{2}}|g_{\phi KK}| \approx 3.3$ should be used. The coefficients A_i, $i=1...6$ and $B_{2,5,6}$ result from the integration over intermediate state D^+D^- and look as

$$\begin{aligned}
A_1 &= \frac{|\boldsymbol{q}_{D^+}|^2}{8\pi s^{1/2}|\boldsymbol{q}^+|}\int_{-1}^{1}x\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx \frac{\lambda_D|\boldsymbol{q}_{D^+}|^3}{6\pi s^{1/2}}\times\exp\left(-\frac{1}{2}\lambda_D s\right), \\
A_2 &= \frac{|\boldsymbol{q}_{D^+}|^3}{16\pi s^{1/2}}\int_{-1}^{1}(1-x^2)\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx \frac{|\boldsymbol{q}_{D^+}|^3}{12\pi s^{1/2}}\times\exp\left(-\frac{1}{2}\lambda_D s\right), \\
A_3 &= -\frac{|\boldsymbol{q}_{D^+}|^3}{4\pi s^{1/2}}\int_{-1}^{1}x(a_D+x)\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx -\frac{|\boldsymbol{q}_{D^+}|^3}{6\pi s^{1/2}}\times \\
&\quad \exp\left(-\frac{1}{2}\lambda_D s\right)\times\left(1-\frac{1}{2}\lambda_D s\right), \\
A_4 &= -\frac{|\boldsymbol{q}_{D^+}|}{16\pi s^{1/2}|\boldsymbol{q}^+|^2}\int_{-1}^{1}\frac{x}{a_D+x}\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx \frac{2|\boldsymbol{q}_{D^+}|^3}{3\pi s^{5/2}}\times \\
&\quad \exp\left(-\frac{1}{2}\lambda_D s\right)\times\left(1+\frac{1}{2}\lambda_D s\right), \\
A_5 &= -\frac{|\boldsymbol{q}_{D^+}|^2}{32\pi s^{1/2}|\boldsymbol{q}^+|}\int_{-1}^{1}\frac{1-x^2}{a_D+x}\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx \frac{|\boldsymbol{q}_{D^+}|^3}{6\pi s^{3/2}}\times \\
&\quad \exp\left(-\frac{1}{2}\lambda_D s\right), \\
A_6 &= -\frac{|\boldsymbol{q}_{D^+}|^3}{32\pi s^{1/2}|\boldsymbol{q}^+|^2}\int_{-1}^{1}\frac{x(1-x^2)}{a_D+x}\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx \frac{2|\boldsymbol{q}_{D^+}|^5}{15\pi s^{5/2}}\times \\
&\quad \exp\left(-\frac{1}{2}\lambda_D s\right)\times\left(1+\frac{1}{2}\lambda_D s\right), \\
B_2 &= \frac{|\boldsymbol{q}_{D^+}|^3}{16\pi s^{1/2}}\int_{-1}^{1}(3x^2-1)\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx \frac{\lambda_D^2|\boldsymbol{q}_{D^+}|^5 s^{1/2}}{60\pi}\times \\
&\quad \exp\left(-\frac{1}{2}\lambda_D s\right), \\
B_5 &= -\frac{|\boldsymbol{q}_{D^+}|^2}{32\pi s^{1/2}|\boldsymbol{q}^+|}\int_{-1}^{1}\frac{3x^2-1}{a_D+x}\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx -\frac{4|\boldsymbol{q}_{D^+}|^5}{15\pi s^{7/2}}\times \\
&\quad \exp\left(-\frac{1}{2}\lambda_D s\right)\times\left[1+\frac{1}{2}\lambda_D s+\frac{1}{2}\times\left(\frac{1}{2}\lambda_D s\right)^2\right], \\
B_6 &= -\frac{|\boldsymbol{q}_{D^+}|^3}{32\pi s^{1/2}|\boldsymbol{q}^+|^2}\int_{-1}^{1}\frac{x(5x^2-3)}{a_D+x}\exp[2\lambda_D(a_D+x)|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|]dx \approx \frac{8\lambda_D|\boldsymbol{q}_{D^+}|^7}{35\pi s^{5/2}}\times \\
&\quad \exp\left(-\frac{1}{2}\lambda_D s\right)\times\left(1+\frac{1}{4}\lambda_D s\right).
\end{aligned} \tag{A.6}$$

Here

$$a_D = \frac{2m_\rho^2-s}{4|\boldsymbol{q}_{D^+}||\boldsymbol{q}^+|}, \tag{A.7}$$

and the approximate equality of the slopes of the D and D^* exchanges $\lambda_{D^*} \approx \lambda_D$ is supposed. The approximate equalities in Eq. (A.6) are valid in the threshold situation. One should have in mind that $m_{D^*} \equiv m_{D^{*0}}$, $m_D \equiv m_{D^+}$ in Eqs. (A.4), (A.6). To obtain the contribution of the intermediate state $D^0\bar{D}^0$ one should make the replacement $m_{D^*} \to m_{D^{*+}}$, $m_D \to m_{D^0}$ in the above expressions. To get the final expressions, one should take, respectively, the sum (difference) of the D^+D^- and $D^0\bar{D}^0$ contributions in the case of the conservation (violation) of isospin in the course of the decay.

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